MOISTURE-RETAINING CAPACITY AND STRUCTURE OF POROUS BODIES

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UDC 539.217.1+532.2

The moisture-retaining capacity $p(\omega)$ of nonswelling porous bodies is explained from the point of view of the statistical geometry of the pore space, and on the basis of the resulting representation of $p(\omega)$ it is shown that it is possible to obtain the distributions of pores with respect to the radii and their derivative.

In soil physics, construction engineering, and related fields, one of the important characteristics of porous bodies is their ability to retain moisture when subjected to suction pressure [1]; however, up to the present time we have no theoretical explanation of the results obtained in such experiments. An attempt to find such an explanation is made in the present study on the basis of a previously developed statistical approach to the description of the structure of porous bodies and the equilibrium of liquid in them [2, 3].

The porous body is assumed to be nonswelling, homogeneous, and isotropic; the pores in it may intersect arbitrarily and the variation of the pore radius along the pore axis is regarded as the realization of a stationary random process.

A porous body completely saturated with liquid is situated in the layer $0 \le z \le h$; below this, at $z < 0$, there is a fine-pore filter and liquid (Fig. 1). The filter is able to withstand negative pressure drops from 0 to p_m, and therefore for suction pressures in the range $0 < p < p_m$ the liquid level remains constant and does not separate from the porous body. The variation of moisture content with pressure is known for the filter; this makes it possible to plot the curves of moisture-retaining capacity from the porous bodies being tested.

As the suction pressure increases, there is drainage of the liquid from the porous body. In [3] it is shown that in the description of the equilibrium distributions of liquid in porous bodies which are achieved in the drainage process, it is more convenient to operate with voids instead of filled pores.

We denote by $w_p(z)$ the equilibrium volumetric moisture content at level z for a suction pressure p in the absence of evaporation. At level $z = h$ the liquid can remain only in those pores in which $p + \rho gh$, $< 2\sigma\cos\theta/r$ or $r < a/p + \rho gh$ and consequently

$$w_p(h) = F\left(\frac{a}{p + \rho gh}\right)$$

or

$$u_p(h) = 1 - F\left(\frac{a}{p + \rho gh}\right), \tag{1}$$

where $u_p(z) = 1 - w_p(z)$ is the probability of finding an empty pore at level z.

We shall assume to start with that the pores do not intersect. In the porous body we single out a thin layer from z to $z - dz$. It is not difficult to show [2] that the increment $u_p(z)$ in this layer has the form

$$-du_p(z) = -\lambda_p(z)\, dz u_p(z), \tag{2}$$

Scientific-Research Institute of Structural Physics, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 28, No. 4, pp. 681-684, April, 1975. Original article submitted September 24, 1975.

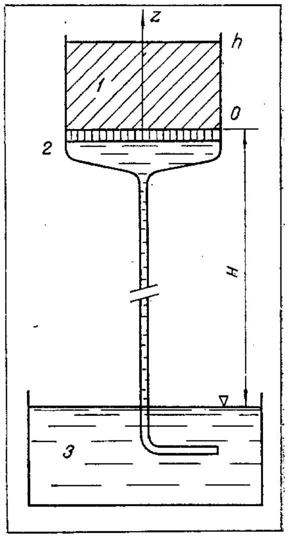

Fig. 1. Scheme of the experiment: 1) porous body being tested; 2) fine-pore filter; 3) liquid, p = ρgH.

where $\lambda_p(z)dz$ is the probability that $r(z-dz) < R(z-dz)$ when $r(z) > R(z)$, i.e., $\lambda_p(z)dz = P[r(z-dz) < R(z-dz) \; r(z) > R(z)]$. Calculations analogous to those of [2] give us

$$\lambda_p(z) = \beta \frac{f(R(z))\,\Gamma_p(z)}{1-F(R(z))}\,; \tag{3}$$

here $\Gamma_p(z) = \int\limits_{-\mu(z)}^{\infty} [\mu(z) + r']\varphi(r')dr'$. Thus, from (2) we obtain a first-order differential equation

$$-\frac{du_p(z)}{dz} = -\lambda_p(z)\,u_p(z) \tag{4}$$

with the boundary condition (1). It is obvious that branching of the pores leads to an increase in the number of empty pores and, consequently, in the probability of finding them, $u_p(z)$. Detailed consideration of the elementary events taking place with empty pores in the layer $(z, z-dz)$ shows [3] that Eq. (4) becomes the more complicated equation

$$-\frac{du_p(z)}{dz} = -\lambda_p(z)\,u_p(z) + \bar{\lambda}_p(z)\,P_p(z). \tag{5}$$

Here $\bar{\lambda}_p(z)dz$ is the probability of variation of the radius of the pore in the layer

$$(z,\; z-dz) \text{ from } r(z-dz) > R(z-dz) \quad\text{to}\quad r(z) < R(z),$$

$P_p(z)$ is the probability that a void will be found in such a place. The functions $\bar{\lambda}_p(z)$ and $P_p(z)$ may be calculated by using a method analogous to that of [3]:

$$\bar{\lambda}_p(z) = \beta f(R(z))\,[\Gamma_p(z) - \mu(z)]/F(R(z)) > 0,$$

$$P_p(z) = \exp\left\{-\int\limits_z^h [\lambda_p(y) + \nu_0\beta/3\cdot(1 - F(R(y)))\,w_p(y)]\,dy\right\}.$$

We define the average moisture content of the porous body, $\omega_h(p)$, as

$$\omega_h(p) = \frac{1}{h}\int\limits_0^h w_p(y)\,dy. \tag{6}$$

The inverse relationship $p = p(\omega)_h$ is usually called [1] the moisture-retaining capacity of the porous body. The volume of the liquid $V(p)$ which has flowed out of the porous body as the suction pressure varies from 0 to p is

$$V(p) = mSh - mS\int\limits_0^h w_p(y)\,dy = mSh\,[1 - \omega_h(p)]$$

or $\omega_h(p) = 1 - V(p)/mSh$. Consequently the average moisture content is expressed in terms of $V(p)$, which can be measured experimentally.

Thus, we have shown that in order to explain the moisture-retaining capacity of a porous body it is necessary and sufficient to know the distribution functions for the pores with respect to the radii and their derivative; in addition, $p = p(\omega_h)$ will depend essentially on h, the thickness of the layer of the porous body.

We write $L = \Omega^{1/3}$, where Ω is the characteristic volume of the porous body, defined in [4]. Now suppose that $h \ll L$; in this case we can neglect the approach of voids to level z from below, i.e., we can neglect the term $P_p(z)$ in Eq. (5). Thus, in the case $h \ll L$ Eq. (4) is valid; solving it, we obtain

$$w_p(z) = 1 - [1 - F(R(h))]\exp\left\{-\int\limits_z^h \lambda_p(y)\,dy\right\}.$$

In this case we find from (6), to within terms of first order in h, that

$$\omega_h(p) = F\left(\frac{a}{p}\right) + \left[\beta \frac{f\left(\frac{a}{p}\right)}{1-F\left(\frac{a}{p}\right)} \Gamma(p)\left(1 - F\left(\frac{a}{p}\right)\right) - f\left(\frac{a}{p}\right)\frac{a\rho g}{p^2}\right]h, \quad (7)$$

where $\Gamma(p) = \Gamma_p(0)$.

From (7) we find that $\partial\omega_h(p)/\partial h = K(p)$ is independent of h. Consequently

$$F\left(\frac{a}{p}\right) = \omega_h(p) - K(p)h. \quad (8)$$

Hence

$$f\left(\frac{a}{p}\right) = -\frac{p^2}{a}[\omega_h' - K'h], \quad \omega_h' = \frac{d\omega_h(p)}{dp}. \quad (9)$$

Comparing (7) and (9), we find that

$$\Gamma(p) = \frac{-\omega_h'\rho g/p^2 + a/p^2 \cdot K(p)}{-\beta\omega_h'} = M(p). \quad (10)$$

Since $w_p(z) = w(p + \rho gz)$, it follows that $dw_p(z)/dp = \rho g\, dw_p(z)/dz < 0$; from this and (6) it follows that $\omega_h' < 0$, and therefore $M(p) > 0$. It is easy to verify that

$$\Gamma'(p) = -\frac{2a\rho g}{p^3}\left[1 - \Phi\left(-\frac{a\rho g}{p^2}\right)\right],$$

or, from (10)

$$1 - \Phi\left(-\frac{a\rho g}{p^2}\right) = -\frac{p^3}{2a\rho g}M'(p) = Q(p).$$

Making use of the fact that $\varphi(r') = \varphi(-r')$, we finally obtain

$$\Phi\left(\frac{a\rho g}{p^2}\right) = Q(p).$$

Thus, measuring the moisture-retaining capacity of a porous body for two values h and h_1 such that $h - h_1 \ll h$ and $h \ll L$, we can obtain the distributions of pores with respect to the radii and their derivative.

It has usually been assumed that for a description of the structure of a pore space it is sufficient to know the distribution of the pores with respect to the radii; as has been shown earlier [2, 3] and in the present study, the distribution of pores with respect to the rate of variation of the radii plays an equally important role. This recalls the situation in radio engineering in which the signals are characterized not only by the distribution with respect to amplitude but also by the distribution with respect to frequencies.

NOTATION

$r(z)$	is the radius of capillary at level z;
σ	is the surface tension;
θ	is the boundary angle;
ρ	is the density of liquid;
$F(r)$	is the pore distribution function with respect to radii;
β	is the tortuosity of the pores;
g	is the acceleration of gravity;
m	is the porosity;
S	is the surface area of the porous body;
$a = 2\sigma\cos\theta$; $\Phi(r')$	is the pore distribution function with respect to the derivative of the radius;
$1/\nu_0$	is the average distance between branchings of pores;

$\varphi(r') = \Phi'(r')$; $f(r) = F'(r)$; $R(z) = a/(p+\rho gz)$; $\mu(z) = a\rho g/(p+\rho gz)^2$.

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